Acoustic Propagation in Very Shallow Water

Michael B. Porter
Heat, Light, and Sound Research
La Jolla, U.S.A.
mikeporter@hlsresearch.com

Martin Siderius
Portland State University
Portland, U.S.A.
siderius@pdx.edu

Abstract—**In the 1970's and 80's there was a great deal of progress in computational ocean acoustics, largely oriented to passive sonar applications at frequencies below 1 kHz. This led to a variety of propagation models based on four principal approaches: 1) normal modes, 2) ray/beam tracing, 3) parabolic equations, and 4) wavenumber integration. In this paper we discuss the application of these techniques to very shallow water, of interest for applications in waterside security. As a particular application of interest, we consider acoustic modems which may be used to provide communications links with underwater vehicles. We discuss the basic physics, the modeling approaches, and the implications for modem performance.**

Keywords: Underwater acoustics, modems, high-frequency

I. Introduction

Sound propagation in the ocean is usually modeled using the acoustic wave equation, which expresses a balance between the spatial and temporal curvature of the acoustic pressure field, p(x, y, z, t):

$$p_{tt} = c^2(x, y, z)\ \nabla^2 p$$

We may draw an analogy to a trampoline whose displacement corresponds to the pressure. The sound speed is a sort of spring constant which attempts to return the curved trampoline surface back to its equilibrium position.

With modern finite-element methods, or traditional finite-difference methods, one may create a discretized version governing the pressure field on a volume of nodes in space. This in turn may be solved on a digital computer. However, as of this writing (and for the last many decades) it has not generally been practical to solve this wave equation in the ocean. This stems from the fact that typical problems are 10-100 wavelengths in depth and 100-1000 wavelengths in range. At some future point this comment will have a rustic sound; however, assuming perhaps 10 points per wavelength are necessary to sample the field, one finds that the volume of sample points is excessive in terms of storage and computation. Therefore a variety of approximations are invoked.

In a somewhat perilous exercise we attempt to summarize the commonly used approximations as follows. First, one almost always assumes that one may model 3D propagation independently along each bearing, ignoring refraction in the lat/lon plane. If the bottom is reasonably flat and the ocean temperature is stratified (to the best of our knowledge) then we can use the mathematical technique of Separation of Variables to split the range-depth problem into independent problems for range and depth. This leads to normal mode expansions and wavenumber integration methods. These two methods are intimately related. The normal modes are the vibrational modes of the oceanic 'trampoline'. Normal mode methods are usually efficient for ranges beyond 10 water depths. Otherwise wavenumber integration often works better.

If range-dependence of the environment is a concern then parabolic equation models provide the most accurate solution. However, ray/beam methods also allow range-dependence and typically provide adequate accuracy with much reduced computational effort. Ray/beam methods are particularly attractive for broadband problems since they allow complicated waveforms to be propagated by simply summing the echoes, taking into account the echo strength and the delay. The other methods mentioned above generally assemble broadband waveforms via Fourier synthesis. This means that the wave equation is solved independently for each tone in the waveform and the tones are summed. Solving the wave equation for each tone is usually a lot of work.

Ray/beam methods fell out of favor in the 70's and 80's as the so-called full-wave methods (modes, PE, wavenumber integration) were favored based on considerations of accuracy. However, there have been significant advances in ray/beam methods in the last 20 years that are not widely appreciated. In our organization we use ray/beam methods for perhaps 90% of the ocean acoustic modeling problems.

Software for these various models is freely available on the Internet. See for instance, http://oalib.hlsresearch.com/.

II. Physics of Sound Propagation in Very Shallow Water

We take as one example some data collected during the ForeFront Experiment on the New England Shelf as indicated in Fig. 1. This was one of many tests done to understand environmental effects on acoustic modems. The water depth is about 25 m at the location of a fixed receiver. A variety of modem waveforms and channel probes were transmitted in the 8-16 kHz band. Before looking at the acoustic data, we consider the roughly 10 cm wavelength, 1-m wave height, and rough bottom. We ask the following questions: will the propagation physics be understandable in terms of discrete echoes, i.e. ray theory? Will surface and bottom interaction immediately scatter the wavefronts? If one interaction causes the wavefronts to break up a little, will the wavefronts be completely diffuse after many interactions?

The propagation physics is suggested by the ray trace in Fig. 2. We see a mixture of purely refracted paths that do not interact with the boundaries, as well as additional bottom reflected paths. There are also surface reflected paths that would be seen if we had allowed steeper take-off angles.

To estimate the Green's function of the ocean channel, i.e. the channel impulse response, we transmitted LFM chirps in the 8-16 kHz band. These were repeated roughly every 200 msecs as the source drifted away in range from the fixed receiver. Following the standard matched-filter process we then correlate the received sound with a replica of the transmitted waveform. The transmitted waveform produces a correlation peak when it aligns with an echo as shown in Fig. 3. The drift starts at about 500 m (top of the figure) from the fixed receiver proceeding over about 6 hours to a range of 10 km at the bottom of the figure. There is a great deal of information about the propagation physics in this plot. We can see that close to the receiver there are a number of clear, distinct echoes, with later echoes becoming more diffuse. As we proceed in range, the multipath spread increases as more echoes are generated. However, the first echoes are actually compressed in the sense that they arrive more closely in time. As time proceeds and the range separation increases further, the multipath spread starts to decrease again.

This summary answers to several questions raised above: there is a clear multipath structure; the echoes remain distinct over many boundary interactions but do tend to become more diffuse with each reflection. The multipath *tends* to have a maximum spread at an intermediate range.

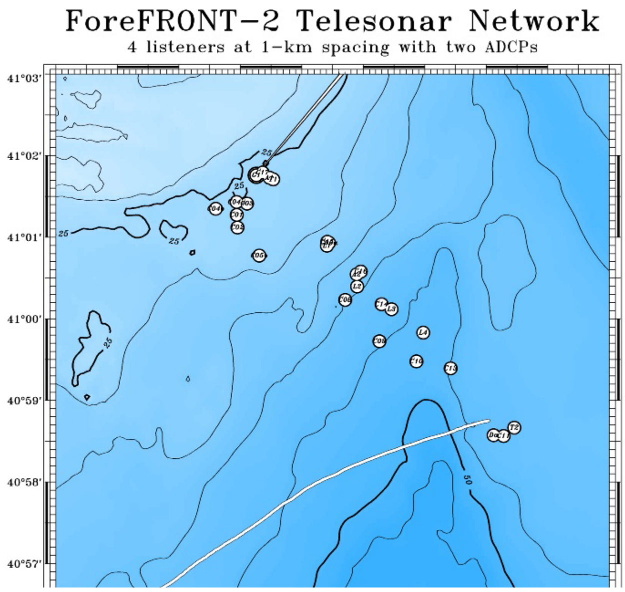

Figure 1. ForeFRONT experiment site, showing the drift track.

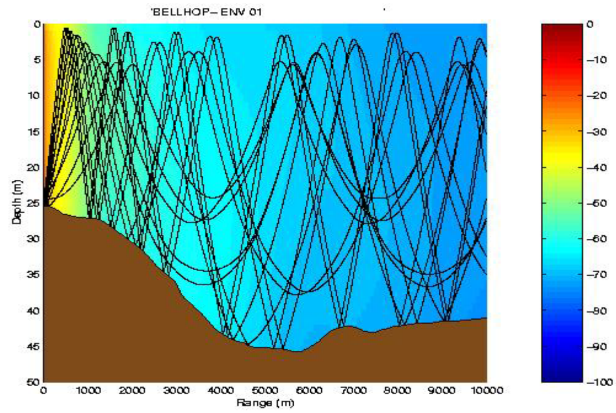

Figure 2. Ray trace superimposed on incoherent transmission loss for a 12-kHz source.

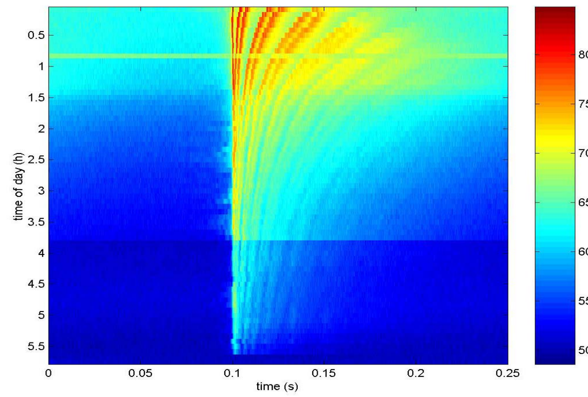

Figure 3. Estimate of the channel impulse response as a function of hours into the experiment. This latter, vertical axis, corresponds to ranges of between 500 m and 10 km.

III. ENVIRONMENTAL EFFECTS ON MODEM PERFORMANCE

As an example of how the propagation conditions may affect system performance in very shallow waters, we consider some modem tests done in the Gulf of Mexico, near Panama City, Florida. The experiment location is shown in Fig. 4. The water depth in this case is roughly constant at about 20 m. An FSK modulation scheme based on the WHOI micromodem was employed with a variety of fixed separations between modems.

The obvious thought would be that one should expect performance, measured in terms of the bit-error rate, to steadily get worse as the modem separation increases. The actual results are shown in Fig. 5. The vertical axis shows the bit-error rate and the horizontal axis shows the SNR at the receiver. The noise figure includes the effects of inter-symbol interference due to multipath.

There are several points of interest in these plots. First, there is no clear trend in performance as a function of the separation between the transmit and receive modems. Generally one would say that the performance is independent of range. However, at sufficiently great separation, the comms link will obviously fail. Second, the performance curves fit neatly on a theoretical curve based on Rician fading. From this, one might think that the performance could be calculated without any knowledge of the propagation physics. However, this is incorrect. The SNR used to place the points on the horizontal axis is a function of the multipath interference, which must either be modeled, or derived from the measured data.

To understand these performance curves, one must look more closely at the propagation conditions. Again we apply matched-filter processing to estimate the channel impulse response. The results measured on a vertical line array for two fixed modem separations are shown in Fig. 6. Note that the multipath spread is greatly reduced at the longer ranges. This is the same stripping of steep angle paths that we observed previously in the ForeFront data. Since these results have been normalized, one cannot deduce the actual transmission loss from them. However, it is a fact that the stripping of the high-angle paths results in significantly higher TL.

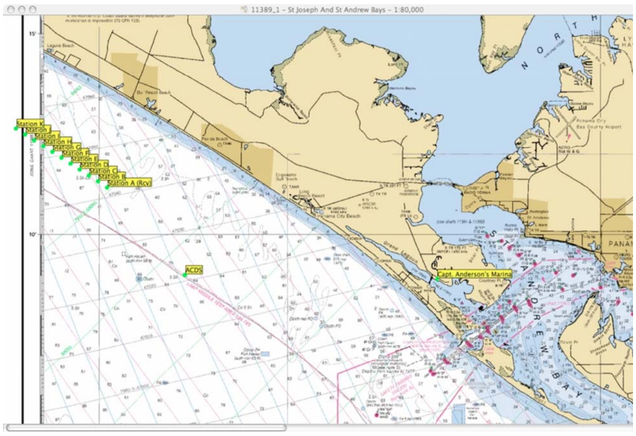

Figure 4. Experiment site off Panama City, Florida.

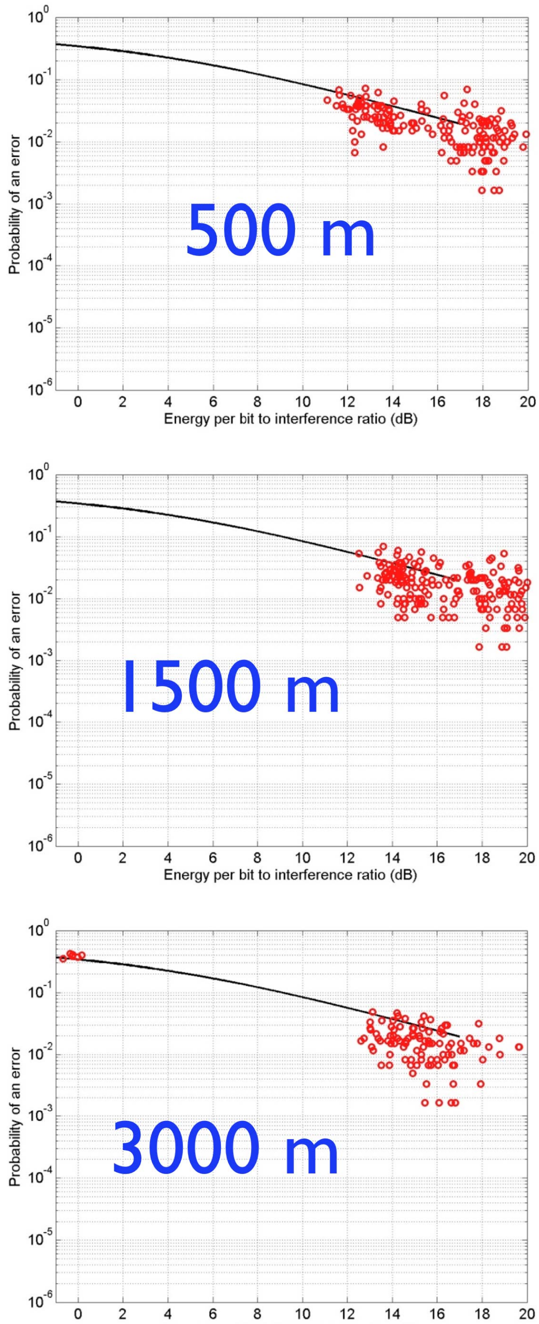

Figure 5. Probability of an error vs. energy by bit to interference ratio. Numbers are presented with out any error correcting codes.

In short, longer ranges have much worse TL; however, that results from the stripping of higher-angle paths which were only interfering with the modems ability to decode the dominant arrival.

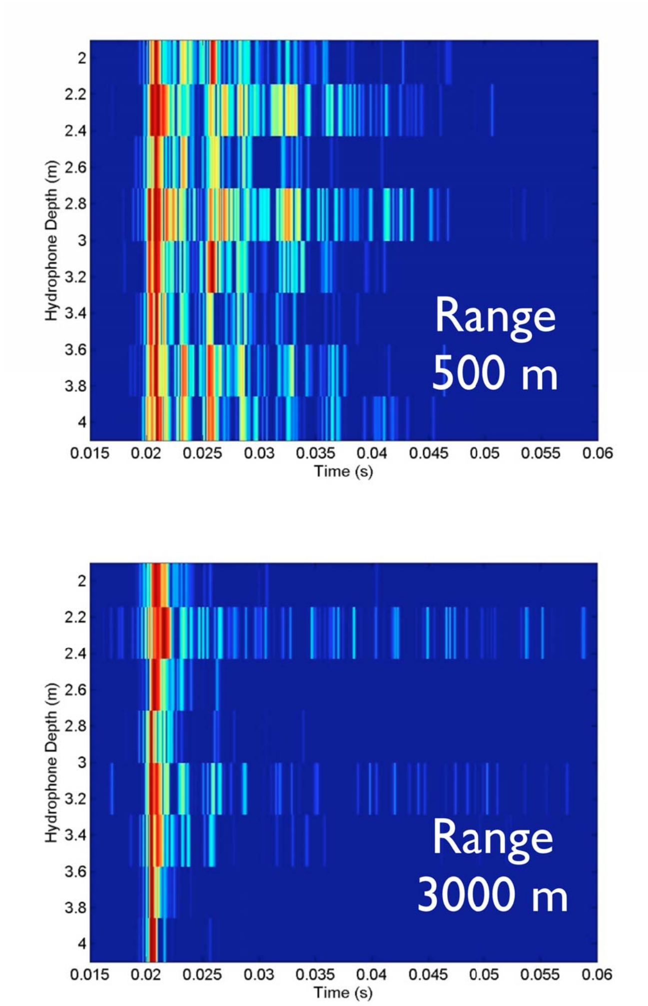

Figure 6. Multipath spread in AUVFest measured on a vertical array and with the transmit modem at ranges of 500 and 3000 m.

IV. SUMMARY

The key points we have tried to convey in this paper are as follows. Models used for low-frequency, deep-water propagation are applicable to the very shallow water conditions of interest for waterside security applications. Roughly speaking, the very shallow water problems are simply scaled versions of traditional passive sonar problems. Performance of a variety of acoustic systems is governed in a subtle way by the propagation physics that such models are designed to study.

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