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THREE-DIMENSIONAL SOUND PROPAGATION IN AN OCEAN OVERLYING AN ELASTIC BOTTOM

GREGORY J. ORRIS, MICHAEL D. COLLINS
Naval Research Laboratory
Washington, DC 20375, USA

GRANT B. DEANE
Scripps Institution of Oceanography
La Jolla, California 92093, USA

and

MICHAEL B. PORTER
New Jersey Institute of Technology
Newark, New Jersey 07102, USA

ABSTRACT The adiabatic mode parabolic equation is generalized to the case of an ocean overlying an elastic bottom. This three-dimensional model is valid when the medium varies sufficiently gradually with the horizontal coordinates so that both coupling of energy between modes and the azimuthal component of displacement may be neglected. The efficiency of the model is demonstrated by applying it to solve a global-acoustics problem involving diffraction by the Hawaiian Islands.

1. Introduction

There has been relatively little three-dimensional propagation modeling in ocean acoustics [1–3] because three-dimensional calculations have been widely regarded as impractical and unnecessary. Many ocean acoustic propagation problems may be solved accurately with the uncoupled azimuth approximation [4], which is based on neglecting the term in the wave equation that involves azimuthal derivatives. This is an important approximation because it is rarely practical to solve three-dimensional problems. Interest in three-dimensional modeling is currently increasing because of a wide interest in shallow water acoustics.

The adiabatic mode parabolic equation (PE) [5] is a three-dimensional propagation model that is practical for solving many problems of interest. This approach is based on the adiabatic mode solution [6], which arises from the assumption that energy coupling between the modes is negligible, and the parabolic equation method [7,8], which is an efficient approach for solving range-dependent problems in ocean acoustics [9,10]. The adiabatic mode PE has been used to solve global-scale problems at low frequencies [11]. This approach has been generalized to handle the effects of fluid flow and applied to model acoustic propagation from the impact sites of the fragments of Comet Shoemaker-Levy 9 with Jupiter [12].

In this paper, we extend the adiabatic mode PE to handle problems involving elastic ocean bottoms. The PE method was previously extended to elastic media for two-dimensional (range and depth) problems [13–16]. In Section 2, we derive the adiabatic mode PE for problems involving an ocean overlying an elastic ocean bottom. The elastic parameters may vary piece-wise continuously in depth. Horizontal variations in the elastic parameters must be sufficiently gradual so that mode coupling and the azimuthal component of displacement may be neglected. In Section 3, we apply the adiabatic mode PE to a global-scale test problem involving diffraction by the Hawaiian Islands. We compare the coupled and uncoupled azimuth solutions to illustrate the importance of three-dimensional effects.

2. The Adiabatic Mode Parabolic Equation

In this section, we describe the adiabatic mode solution for sound propagation in an ocean overlying an elastic bottom. We work in cylindrical coordinates, where z is the depth below the ocean surface, r is the horizontal distance from a time-harmonic point source of circular frequency ω, and θ is the azimuth. The spatially varying parameters are the compressional speed c_p, the shear speed c_s, the density ρ, the compressional attenuation β_p, and the shear attenuation β_s. In the water column, $c_s = \beta_p = \beta_s = 0$ and $\rho = 1$ g/cm^3.

We remove the time-dependent factor $\exp(-i\omega t)$ and the cylindrical spreading factor $r^{-1/2}$ from the dependent variables. We assume that horizontal variations in the elastic parameters are sufficiently gradual so that energy does not couple between modes and that the azimuthal component of displacement is dominated by the other components. Under these assumptions, the following equations of motion are valid in the farfield [16]:

$$(\lambda + 2\mu)\frac{\partial^2 \Delta}{\partial r^2} + (\lambda + 2\mu)\frac{1}{r^2}\frac{\partial^2 \Delta}{\partial \theta^2} + \frac{\partial}{\partial z}\left[(\lambda + 2\mu)\frac{\partial \Delta}{\partial z}\right] + \rho\omega^2\Delta +$$
$$2\frac{\partial \mu}{\partial z}\frac{\partial^2 w}{\partial r^2} + \omega^2\frac{\partial \rho}{\partial z}w + \frac{\partial}{\partial z}\left(\frac{\partial \lambda}{\partial z}\Delta + 2\frac{\partial \mu}{\partial z}\frac{\partial w}{\partial z}\right) = 0 \ , \tag{1}$$

$$\mu\frac{\partial^2 w}{\partial r^2} + \mu\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \mu\frac{\partial^2 w}{\partial z^2} + \rho\omega^2 w + (\lambda + \mu)\frac{\partial \Delta}{\partial z} + \frac{\partial \lambda}{\partial z}\Delta + 2\frac{\partial \mu}{\partial z}\frac{\partial w}{\partial z} = 0 \ , \tag{2}$$

where w is the vertical displacement, Δ is the divergence of the displacement vector, and the complex Lamé parameters λ and μ are defined in [16]. These equations are valid for elastic layers involving piece-wise continuous depth variations in the elastic parameters. The interface conditions described in [16] are applied to handle problems involving fluid layers. The equations of motion are in the form,

$$\frac{\partial^2}{\partial r^2}\begin{pmatrix}\Delta \\ w\end{pmatrix} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\begin{pmatrix}\Delta \\ w\end{pmatrix} + L^{-1}M\begin{pmatrix}\Delta \\ w\end{pmatrix} = \begin{pmatrix}0 \\ 0\end{pmatrix}, \tag{3}$$

where the matrices L and M contain depth operators.

The normal mode representation of the solution of (3) is of the form,

$$\begin{pmatrix}\Delta \\ w\end{pmatrix} = \sum_n \begin{pmatrix}\Delta_n(z; r, \theta) \\ w_n(z; r, \theta)\end{pmatrix}\psi_n(r, \theta), \tag{4}$$

$$L^{-1}M\begin{pmatrix}\Delta_n \\ w_n\end{pmatrix} = k_n^2(r, \theta)\begin{pmatrix}\Delta_n \\ w_n\end{pmatrix}, \tag{5}$$

where (Δ_n, w_n) and k_n^2 are the modes and eigenvalues [17,18] and ψ_n is to be determined. The semicolons in the arguments of Δ_n and w_n indicate gradual variation with r and θ.

Substituting (4) into (3) and applying the assumption that energy coupling between modes may be neglected, we obtain the adiabatic mode wave equation,

$$\frac{\partial^2\psi_n}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\psi_n}{\partial\theta^2} + k_n^2\psi_n = 0. \tag{6}$$

We factor the operator in (6) into incoming and outgoing operators to obtain

$$\left(\frac{\partial}{\partial r} + i\sqrt{\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + k_n^2}\right)\left(\frac{\partial}{\partial r} - i\sqrt{\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + k_n^2}\right)\psi_n = 0. \tag{7}$$

Assuming that outgoing energy dominates, we obtain the outgoing adiabatic mode wave equation,

$$\frac{\partial\psi_n}{\partial r} = i\sqrt{\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + k_n^2}\psi_n. \tag{8}$$

We rearrange (8) to obtain

$$\frac{\partial\psi_n}{\partial r} = ik_0\sqrt{1+X}\psi_n, \tag{9}$$

$$X = k_0^{-2}\left(\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + k_n^2 - k_0^2\right), \tag{10}$$

where k_0 is a representative horizontal wave number. Substituting a rational approximation for the square root in (9), we obtain the adiabatic mode PE,

$$\frac{\partial\psi_n}{\partial r} = ik_0\left(1 + \sum_{j=1}^{m}\frac{a_{j,m}X}{1+b_{j,m}X}\right)\psi_n. \tag{11}$$

This equation is solved with standard numerical techniques as described in [5]. Choices for the coefficients $a_{j,m}$ and $b_{j,m}$ are given in [16,19].

3. Application to Global Acoustics

There has recently been a great deal of interest in global acoustics [20–22]. To apply the adiabatic mode PE to global-scale problems, we replace the spreading factor $r^{-1/2}$ with $R^{-1/2}$, where

$$R = R_0\sin\left(\frac{r}{R_0}\right), \tag{12}$$

and R_0 is the radius of the earth. The following generalization of (8) was derived in [5] for this case:

$$\frac{\partial\psi_n}{\partial r} = i\sqrt{\frac{1}{R^2}\frac{\partial^2}{\partial\theta^2} + k_n^2}\psi_n. \tag{13}$$

This equation is solved with the same approach that is used to solve (8).

We apply the adiabatic mode PE to a problem involving a 1-Hz source to the southeast of the Hawaiian Islands at 210° E and 15° N. Since the geographic dependence of the sound speed in the ocean is a minor factor for this problem, we use a single profile that is representative of temperate regions. Since we do not have access to a global data base

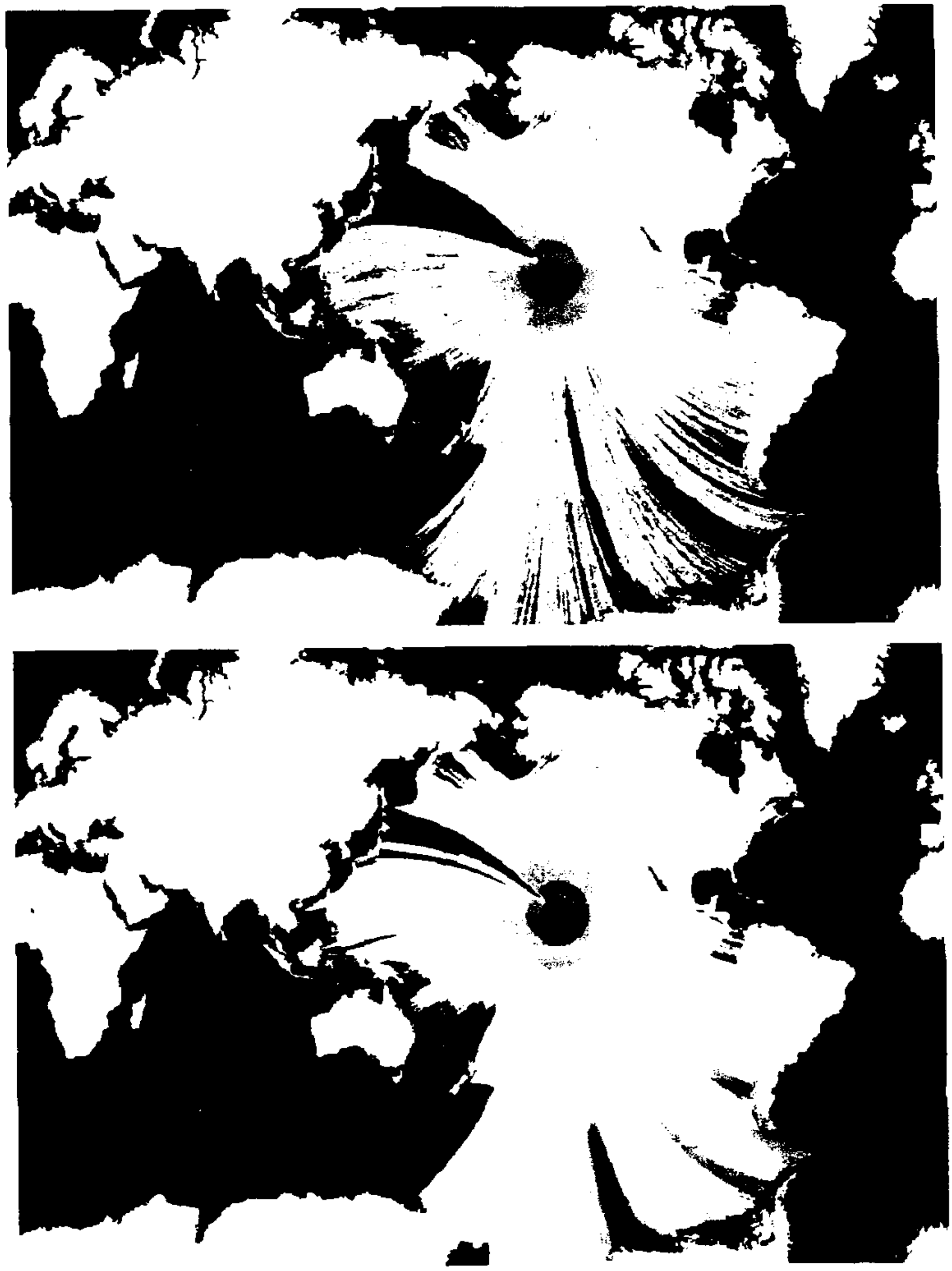

Figure 1. Adiabatic mode PE solutions for the first mode at 1 Hz. Azimuthal coupling is included (top) and neglected (bottom). The dynamic range is 100 dB, with red corresponding to high intensity and blue corresponding to low intensity.

Figure 2. Magnified view of the solutions appearing in Figure 1. Azimuthal coupling is included (top) and neglected (bottom). The dynamic range is 100 dB, with red corresponding to high intensity and blue corresponding to low intensity.

for the parameters of the ocean bottom, we also use constant profiles for these parameters. The wave speeds are linear functions of depth defined by the values $c_p(z_0) = 3400$ m/s, $c_p(z_1) = 6000$ m/s, $c_s(z_0) = 1700$ m/s, and $c_s(z_1) = 3000$ m/s, where $z_0 = 0$ and $z_1 =$ 20 km. The parameters $\rho = 1.5$ g/cm^3, $\beta_p = 0.1$ dB/λ, and $\beta_s = 0.2$ dB/λ are taken to be constants.

Appearing in Figures 1 and 2 are adiabatic mode PE solutions for the first mode that include and neglect azimuthal coupling. The Hawaiian Islands cast a shadow that is broadened by horizontal refraction (an azimuthal coupling effect). Since the horizontal phase speed is larger in shallower water, this is consistent with the ray solution. These solutions are qualitatively similar to the solutions that appear in [11] for the case of a fluid bottom that has a qualitatively similar horizontal phase speed dependence on ocean depth.

The adiabatic mode PE computation required about 5 hours on a Silicon Graphics computer with a 150-MHz MIPS R4400 chip. The computations were done using a three-term rational approximation in the adiabatic mode PE, a range step of 10 km, and 256 azimuthal grid points per degree. The computations were carried out over the entire earth, including the oceans and the continents. The first eigenvalue was obtained as a function of ocean depth using the approach described in [17,18].

4. Conclusions

The adiabatic mode PE has been extended to handle problems involving an ocean overlying an elastic bottom. This technique is valid when mode coupling and the azimuthal component of displacement are negligible and is practical for solving large-scale three-dimensional problems. The efficiency of the model was illustrated by applying it to solve a global acoustics problem involving diffraction around the Hawaiian Islands.

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