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SCATTERING OF WAVES FROM TARGETS EMBEDDED IN A LAYERED MEDIUM

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Abstract: *The use of the virtual source method in scattering from 3D targets in a 3D ocean is discussed. To be able to rapidly produce target echoes over a wide frequency band, a multi-path expansion solution is used to produce the waveguide Green's functions. The technique is applied to calculation of scattering from a sphere, a truncated cone, and a model rock.*

1. INTRODUCTION

Accurate modelling of scattering from a target in a layered medium requires the solution of the wave equation while simultaneously satisfying boundary conditions at all boundaries, including those of the target. In modelling the propagation of waves in a layered medium in the absence of the target, appropriate coordinate systems are used in which the boundary conditions can easily be imposed. However, the presence of the target, except in some highly special cases, makes it difficult to find a coordinate system where the boundary conditions can easily be applied to both the surface of the target and the boundaries of the medium. One way to solve this problem is to use the finite-difference or finite-element methods where boundary conditions are incorporated into a discretized form of the governing equations [1]. However, since the size of the resulting matrices in both methods grows as the size of the region of computation, the numerical cost of solving a scattering problem in an area many thousands of wavelengths in size can quickly become insurmountable. To cope with these numerical difficulties, a single-scatter approximation is commonly used [2] where the scattering and propagation problems are solved separately. However, since in this method boundary conditions are not satisfied on the surface of the target and the boundaries of the medium simultaneously, the single-scatter result does not provide a self-consistent solution.

An exact way to solve the problem of scattering from a target in a layered medium is to use the *method of virtual sources* (also know as the *method of superposition*, the *method of equivalent sources* or the *method of auxiliary sources*). In this method, the target is replaced by a collection of sources, whose complex amplitudes are determined by satisfying the boundary conditions at the surface originally occupied by the target [3, 4, 5]. By choosing these sources to be the medium Green's functions, a self-consistent solution can be obtained that satisfies the wave equation, the radiation condition, as well as the boundary conditions on all surfaces, including the surface of the target. Furthermore, by replacing the target with a collection of sources, the scattering and propagation problems are converted to a multi-source propagation problem, which can be solved by any standard propagation model.

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In this paper scattering from a 3-D target in an oceanic waveguide is modelled using the virtual source technique. To be able to rapidly produce the target echoes, a multi-path expansion is used to compute the waveguide Green's functions. In Section 2 the method is described. In Section 3 the method is applied to scattering from a sphere, a truncated cone, and a model rock (henceforth referred to as the rock), followed by conclusions in Section 4.

2. A MULTI-PATH EXPANSION SOLUTION FOR SCATTERING FROM A 3-D TARGET IN A WAVEGUIDE

In the virtual source formulation, scattering from a general, penetrable elastic target is modelled by using virtual sources to produce the field in the interior and exterior of the target. In most cases, virtual sources of unknown complex amplitudes are placed just inside the target to produce the field in its exterior and just outside to produce the field in its interior. The functional form of the fields in the two regions can be any complete set of basis functions, as long as they satisfy the wave equation. The exterior basis functions must also satisfy the radiation condition. The target (interior) and the ocean (exterior) Green's functions satisfy the above conditions and can be used as basis functions. The interior Green's function expresses the response of the target to a unit point source located in its exterior. The exterior Green's function expresses the response of the surrounding environment to a unit point source. We represent the exterior pressure Green's functions by $G^{(1)}(x,x')$ and the interior pressure Green's functions by $G^{(2)}(x,x')$, where x is the location of the receiver and x' is the location of the source. Then, imposing continuity of pressure and normal displacement at the surface of the target gives:

$$\left.\begin{aligned} &p_{inc}(\mathbf{x}) + G^{(1)}(\mathbf{x},\mathbf{y})\mathbf{S} = G^{(2)}(\mathbf{x},\mathbf{z})\mathbf{Q}, \\ &\mathbf{n}\cdot\nabla p_{inc}(\mathbf{x}) + \mathbf{n}\cdot\nabla G^{(1)}(\mathbf{x},\mathbf{y})\mathbf{S} = \mathbf{n}\cdot\nabla G^{(2)}(\mathbf{x},\mathbf{z})\mathbf{Q} \end{aligned}\right\} \ \mathbf{x}\in\partial V \tag{1}$$

where V and ∂V denote the volume of the target and its boundary, respectively. In addtion p_{inc} represents the incident field, $\mathbf{n}$ is the outward unit normal vector to the surface of the target, the vector $\mathbf{x}$ represents the nodes on the boundary, where the boundary conditions are imposed, and the vectors $\mathbf{y}$ and $\mathbf{z}$ represent the location of sources of amplitude $\mathbf{S}$ and $\mathbf{Q}$, respectively. For the case when the number of sources and nodes are equal, the source amplitudes can be obtained from the above equations:

$$\begin{aligned} &\mathbf{S} = \left(G^{(1)} - \mathbf{K}G_u^{(1)}\right)^{-1}\left(\mathbf{K}\,u_{inc} - p_{inc}\right), \\ &\text{where, } \mathbf{K} = G^{(2)}\left(G_u^{(2)}\right)^{-1},\ u_{inc} = \mathbf{n}\cdot\nabla p_{inc},\ G_u^{(1,2)} = \mathbf{n}\cdot\nabla G^{(1,2)}. \end{aligned} \tag{2}$$

Equation (1) shows the mechanism by which the virtual source technique couples the propagation and scattering problems using the waveguide and the target Green's functions and the boundary conditions. Once the source amplitudes are determined, the propagation and scattering problems are converted to a multi-source propagation problem, which is entirely determined by the waveguide Green's function, $G^{(1)}(x,x')$. Determination of the source amplitudes using the boundary conditions guarantees that the boundary conditions on the surface of the target are satisfied. Since the waveguide Green's functions satisfy the waveguide boundary conditions by construction, a self-consistent solution is obtained that satisfies the boundary conditions everywhere and thus accounts for such effects as multiple scattering, which are neglected in the single-scatter solution.

The target Green's functions can be measured or computed numerically using, for example, the finite element method. The waveguide Green's function can be computed using any propagation model. Our intention in this paper is to be able to rapidly produce the broadband target response in a waveguide over a wide range of frequencies. To do this, we develop a multi-path expansion solution for the waveguide Green's function.

The multi-path expansion is based on the method of images, which treats a source in a waveguide as an infinite sum of image sources [6]. This expansion is exact for perfectly reflecting boundaries, where the reflection coefficients are not functions of angle of incidence. If the boundaries are penetrable and thus the reflection coefficients are functions of angle of incidence, then this expansion becomes a stationary phase approximation, valid at high frequencies, where the reflection coefficients are computed only at specular angles. We consider a waveguide composed of an isovelocity water layer of depth *h* over an isovelocity half-space bottom. If we denote water parameters by subscript 1 and bottom parameters by subscript 2, then field at (r,z) due to a source at $(0,z_s)$ is given by

$$p(r,z) = \sum_{l=0}^{\infty}(-1)^l\left[V^l(\theta_{l_1})\frac{e^{ikR_{l_1}}}{R_{l_1}} + V^{l+1}(\theta_{l_2})\frac{e^{ikR_{l_2}}}{R_{l_2}} - V^l(\theta_{l_3})\frac{e^{ikR_{l_3}}}{R_{l_3}} - V^{l+1}(\theta_{l_4})\frac{e^{ikR_{l_4}}}{R_{l_4}}\right] \quad (3)$$

In the above equation R_{l_i} and the reflection coefficients, $V(\theta_{l_i})$ are given by

$$R_{l_i} = \sqrt{r^2 + z_{l_i}^2},\ i = 1,\ 2,\ 3,\ 4.$$
$$z_{l_1} = 2hl + z_s - z,$$
$$z_{l_2} = 2h(l+1) - z_s - z,$$
$$z_{l_3} = 2hl + z_s + z,$$
$$z_{l_4} = 2h(l+1) - z_s + z,$$

$$V(\theta_{l_i}) = \frac{\dfrac{\rho_2 c_2}{\sqrt{1-(k_1/k_2)^2\cos(\theta_{l_i})^2}} - \dfrac{\rho_1 c_1}{\sin(\theta_{l_i})}}{\dfrac{\rho_2 c_2}{\sqrt{1-(k_1/k_2)^2\cos(\theta_{l_i})^2}} + \dfrac{\rho_1 c_1}{\sin(\theta_{l_i})}},$$

$$\sin(\theta_{l_i}) = \frac{z_{l_i}}{R_{l_i}},$$

$$k_{(1,2)} = \frac{\omega}{c_{(1,2)}}.$$

In the above ρ and c denote the density and the sound speed. Because of the ray nature of the above model, unlike most other propagation models, its execution speed is independent of frequency. This proves advantagious when rapid computation of target responses over a wide range of frequencies is desired. For any given combination of source and receiver, the above model is used to compute the waveguide Green's functions for use in Eq. (1). The results are presented in the next section.

3. RESULTS

Our method of computing scattering from a target in a waveguide using the virtual source method involves three phases. In phase one, the incident field is computed at the nodes on the surface of the target. In phase two, Eq. (1) is used to compute the source amplitudes, and in phase three the target is replaced by the virtual sources and the field due to the true source and the virtual sources are computed at the receivers. The multi-path expansion model described in Section 2 is used to compute the field in all three stages. All the targets used in the examples presented in this paper are non-penetrable and satisfy the Dirichlet boundary condition at their surfaces (the pressure vanishes at the surface). However, it should be noted that treatment of penetrable targets can be achieved in exactly the same manner, using Eq.(1), except in this case the target Green's function, $G^{(2)}(x,x')$, will be obtained from measurement or a finite element model. To be able to adequately represent the field on the surface of the target, at least five sources per wavelength are used. The sources are distributed evenly and placed just inside the target, on a surface, which can be obtained by displacing the original surface by a small amount along its local inward normal. In the examples presented here, we use the same number of nodes as the number of sources. Recall that the nodes are points on the surface, where the boundary conditions are imposed. Figure (1) shows how the sources and the nodes are distributed on the surface of a rock, which is one of the targets used in this paper. The nodes are placed on the centroid of each triangular facet and the sources are placed at a small distance along the inward normal to the facet.

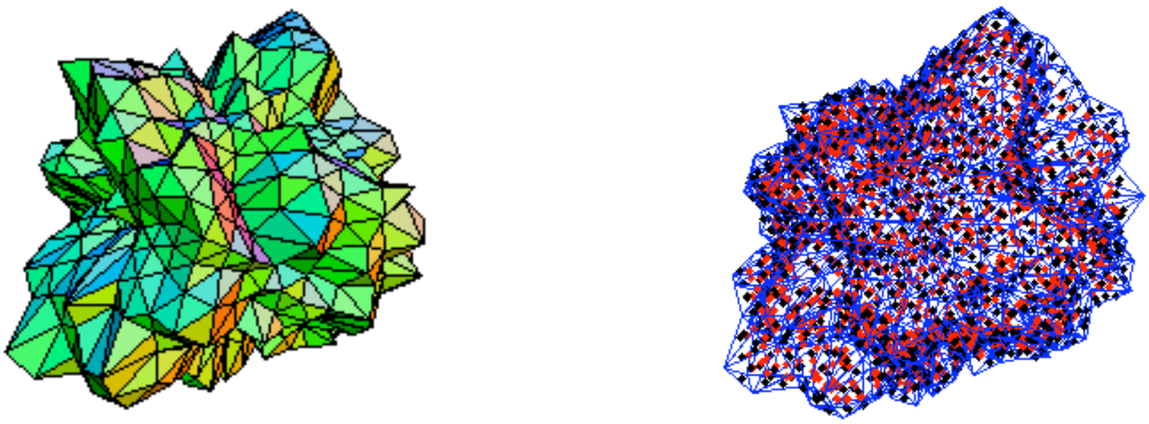

Figure 1: The figure on the left shows the rock used in one of the examples and the figure on the right shows the distribution of sources (red dots) and nodes (black dots) on its surface.

As a first example, we consider 3D scattering from a truncated cone in a Pekeris waveguide, using the multi-path model described in Section 2. The target has a base radius of 1 m, a top radius of 0.5 m and a height of 0.5 m. The bottom sound speed is 1700 m/s and its density is 1.5 g/cc. The water depth is 50 m and the target is placed on the bottom 40 m away from an omni-directional source, which is located at a depth of 10 m. The source frequency is 4 kHz and at this frequency, 700 sources are used. Figure 2 shows the pressure field in the waveguide in the presence of the target in the plane y=0. Because of its small size the target produces a small scattered field. The plot in the bottom is a blow-up of the field in the vicinity of the target. On this scale it is easy to see the scattered field. A quantitative way to test the validity of the solution is to examine whether the boundary conditions are satisfied on the surface of the target. This test is a standard procedure in our analysis and has been performed on all the examples presented here.

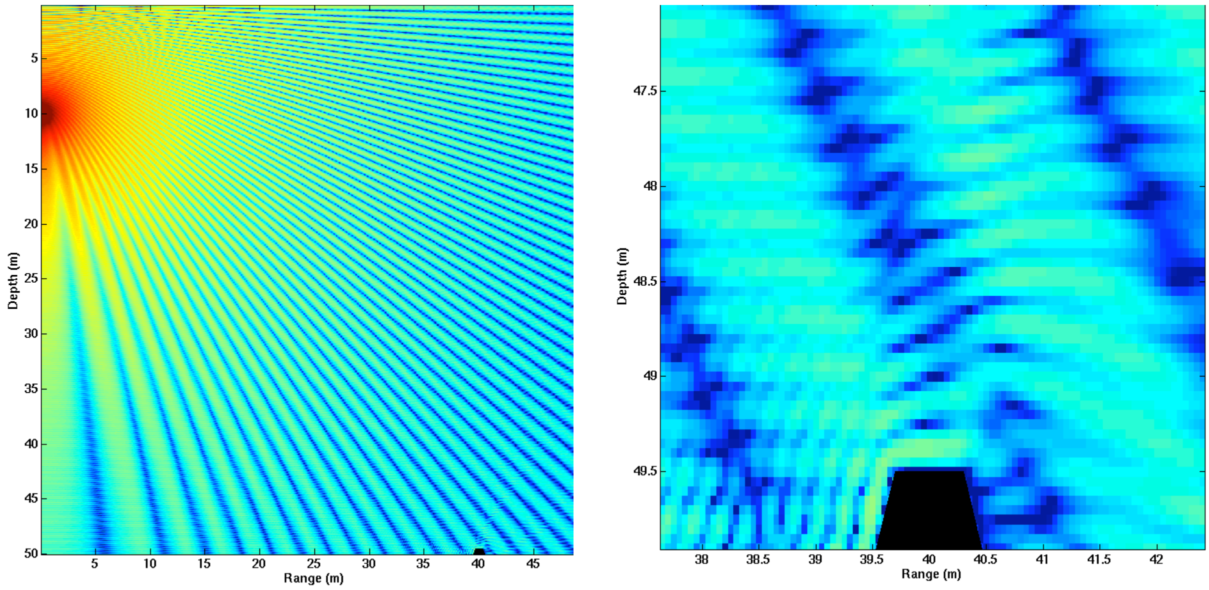

Figure 2: The pressure field scattered by a truncated cone in a waveguide. The right figure is a close-up of the field in the vicinity of the target.

As a second example, we treat scattering from a rigid sphere placed in the same waveguide. The sphere has a radius of 0.5 meters and 900 sources are used in the virtual source model. The sphere is placed at the same location as the truncated cone. The pressure field in the y=0 plane in the vicinity of the sphere is shown in Figure 3, which shows how the presence of the sphere alters the structure of the field. Obviously, the structure of the scattered field in this case is different than that produced by the truncated cone.

As a final example, the right panel in Figure 3 shows the pressure field in the vicinity of the rock shown in Figure 1. The rock is placed at the same location as the previous two targets and 960 sources are used in the virtual source model. And like the other two examples, the field is displayed in the y=0 plane

Figure 4 shows the returned echoes from two of the above targets for a source that transmitted a uniform signal in the 2.5 to 4kHz frequency range. The source is located at a depth of 10 m, the waveguide depth is 30 m and in each case, the targets are placed on the bottom, 30 m away from the source. In all cases, the receiver is located at a range of 20 m and at a depth of 2 m. The top panel in Figure 4 shows the received signal in the absence of the target, where the direct, surface-reflected and bottom-reflected arrivals are observed. The middle panel shows the received signal when the sphere is present. The echo from the target is observed at approximately 32 ms due to the direct arrival and at approximately 42 ms due to the surface-reflected arrival. The bottom panel shows the received signal when the truncated cone is present. Again, the target echo can be observed at approximately 32 ms, but it is weaker and has a different shape than that of the sphere. The echo due to the surface-reflected return is barely noticeable.

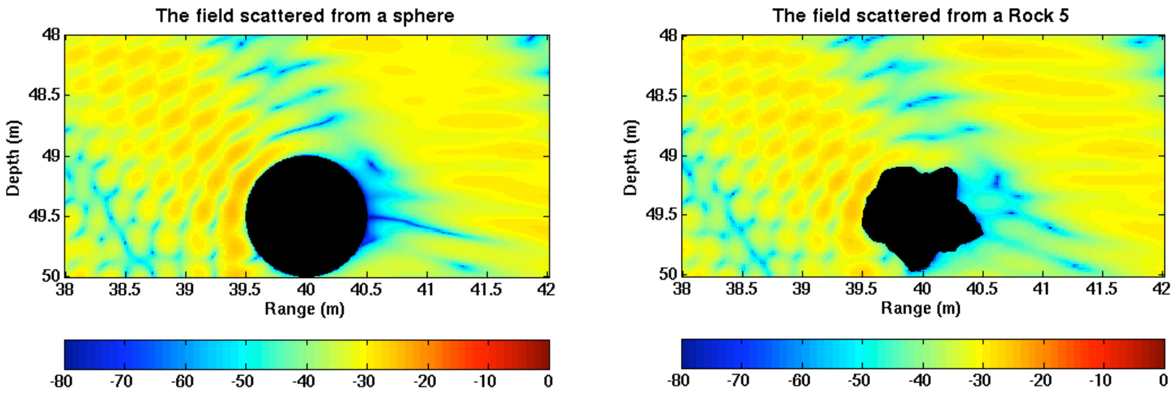

Figure 3: The left panel shows the scattered field in the vicinity of a sphere in a Pekeris waveguide and the right panel shows the same for the rock.

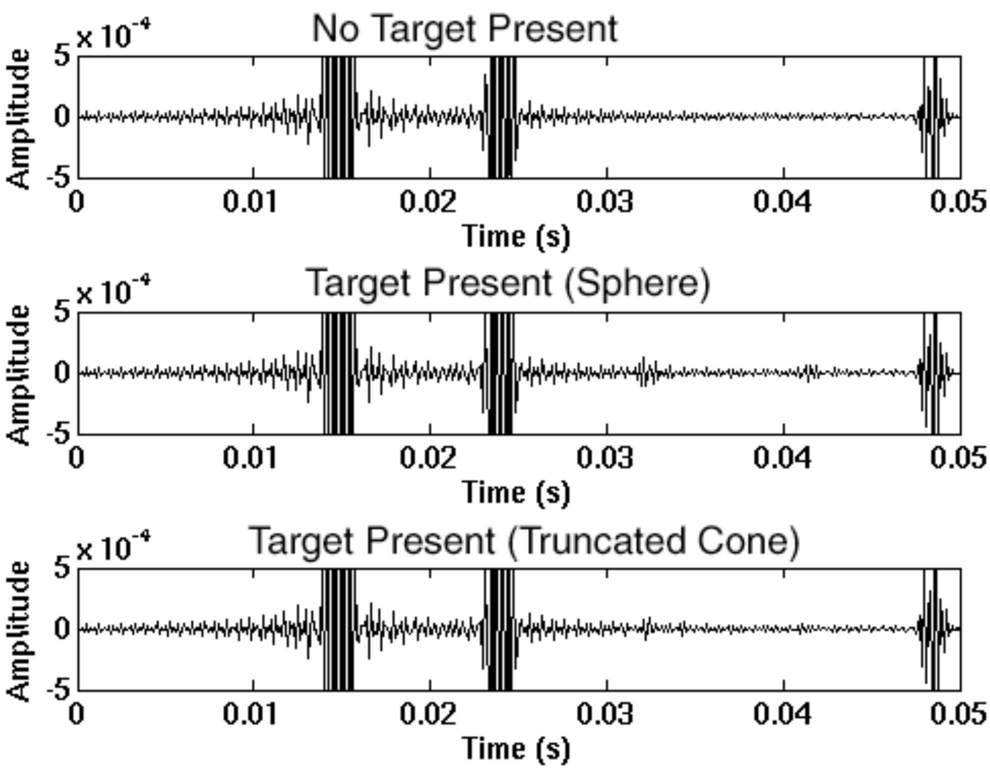

Figure 4: Target echoes in a Pekeris waveguide. The top panel shows the received signal at (r=20 m, z=2 m) in the absence of any target. The middle panel shows the received signal when the sphere is present and the bottom panel shows the same for the rock. Target returns are observed at approximately 32 ms and 42 ms.

4. CONCLUSIONS

The problem of scattering in a layered medium is particularly challenging because it requires the coupling of propagation and scattering. Models such as the normal mode, the parabolic equation, ray theory, and wavenumber integration are well suited for computing wave propagation at long ranges. However, the very approximations that make them so efficient for long-range propagation, make them unsuitable for solving the wave equation in the interior and the vicinity of the target. Similarly, the finite-element technique, which provides accurate solutions of the wave equation in the interior and vicinity target, is not suited for solving the wave equation at long ranges. The virtual source technique provides the means to couple the two methods and thus provide a self-consistent solution for the scattering of waves from targets embedded in a waveguide. The virtual source technique replaces the target by a collection of sources and converts the scattering and propagation problems to a single, multi-source propagation problem. The number of sources that are used in the virtual source technique increases as a function of frequency. Therefore, the next challenge is to develop techniques to expedite the execution speed of the model, particularly at high frequencies and/or when broadband responses of targets are desired. In this paper, a multi-path expansion solution is used to compute the waveguide Green's function. Because of the ray nature of this model, its execution speed is independent of frequency, which is not the case for wave-based propagation models. Although the use of the above model speeds up computation considerably, more improvement in speed can be achieved by using better basis functions and thus reducing the required number of sources and, in parallel, using a multipole expansion to improve propagation speed from the sources to the receivers.

REFERENCES

[1] Burnett, D. S., and Zampolli, M., "Development of a finite-element, steady state, 3D acoustics code for target scattering", SACLANTCEN Report No. SR-379, June 2003.
[2] Ingenito, F. "Scattering from an object in a stratified medium", J. Acoust. Soc. Am., 82 (6), Dec. 1987.
[3] Schmidt, H. "Virtual source approach to scattering from partially buried elastic targets", *In High Frequency Ocean Acoustics*, edited by Michael B. Porter, Martin Siderius and William A. Kuperman, AIP Conference Proceedings 728.
[4] Stepanishen, P. R., "A generalized internal source density method for the forward and backward projection of harmonic pressure fields from complex bodies", J. Acoust. Soc. Am. 101 (6), June 1997.
[5] Ahmad T. Abawi and Michael B. Porter, "The use of the equivalent source technique in calculation of scattering from underwater targets", *In Boundary Influences in High Frequency, Shallow Water Acoustics,* edited by Nicholas G. Pace and Philippe Blondel.
[6] L. M. Brekhovskikh and Yu. P. Lysanov, "*Fundamentals of Ocean Acoustics*", Springer-Verlag, 1991.