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COMPARISON OF BEAM TRACING ALGORITHMS

Paul A. Baxley and Homer Bucker

Space and Naval Warfare Systems Center, San Diego, Acoustic Branch D857,
49575 Gate Road Room 170, San Diego, California, USA 92152-6435
email: baxley@spawar.navy.mil

Michael B. Porter

Science Applications International Corporation
888 Prospect St., Suite 201, La Jolla, California, USA 92037
email: michael.b.porter@saic.com

The method of Gaussian beam tracing has been extensively studied in the last 15 years, especially in seismology. In ocean acoustics there are now four implementations that are superficially similar but with quite different characteristics. The method of Simple Gaussian Beams [Bucker, 1975] expands the beam-width in proportion to the range from the source. In 1984, Porter and Bucker developed a more formal procedure based on work in seismology of Cerveny, et al., that uses differential equations to track the beam spreading. In the late 80's, Porter used the beam structure to develop Geometric Beam Tracing which uses the Gaussian beam equations to calculate the spreading of the ray tube. Finally, in 1990 Weinberg and Keenan developed GRAB (Gaussian Ray Bundles) which is closely related to Geometric Beam Tracing but which limits the beam focusing in a way which seems to handle simple caustics accurately. In this paper, we compare the pros and cons of the different approaches in a variety of ocean scenarios.

1. INTRODUCTION

Ray models are well suited for the prediction of sound fields in range-dependent environments. This is particularly true for high frequency problems, since ray theory represents a high-frequency limit for the wave solution, and wave models can become computationally intensive for such problems. In addition, ray methods are useful for solving broadband problems since many parts of the computation are independent of frequency (e.g. ray paths and travel times).

Nevertheless, ray methods suffer from a number of practical limitations. First, since the intensity in most ray models is calculated from the spreading of two adjacent rays, caustics (points of infinite intensity) appear in regions where ray paths cross or converge. The incorporation of the appropriate phase change at a caustic can be difficult. Second, ray models predict non-physical perfect shadows in regions where rays do not propagate. Third, ray models typically attempt to locate eigenrays (rays that connect source and receiver), which is a difficult nonlinear root-finding problem. This problem becomes particularly worrisome when dealing with propagation in three dimensions. Fourth, most ray models introduce numerical artifacts that decrease the accuracy of the solution compared to ray-theoretic results. Lastly, the ray solution is inherently a high-frequency solution; this precludes its use at low frequencies in complex environments that could profit from its range-dependent features.

Gaussian beam tracing was introduced as a method which retains the good qualities of ray methods, while addressing the above implementation problems. This method consists of approximating a given source by a fan of beams that propagate through the medium according to the standard ray equations. The influence of a beam is defined by a Gaussian distribution, or other symmetric function, centered about that beam. The field at any given point is then constructed by adding up the contribution from each beam at that point. This method therefore does not require eigenray computation. For a Gaussian distribution, perfect shadows and infinite caustics are eliminated, and the applicability to lower frequency problems in improved. In addition, if a triangle distribution is used, the method recovers the ray-theoretic result.

There are four approaches to the Gaussian beam tracing method currently being applied to the ocean acoustics problem. While superficially similar, they differ mainly is the type of beam distribution used (Gaussian or triangle) and the manner in which beam spreading is handled. The most rigorous method is that of Porter and Bucker [1], which is based on the seismological work of Cerveny. et al. [2] In this approach, which will be referred to as the Cerveny Method, Gaussian beam spreading is governed by a pair of differential equations which are integrated along with the standard ray equations. Bucker [3] provides a simpler approach, termed Simple Gaussian Beams (SGB), in which the beam-width expands in proportion to the arc length of the beam path. Porter [4] later used the beam construct to develop Geometric Beam Tracing (GBT), which uses the Cerveny beam equations to calculate the spreading of the ray tube. GBT replaces the Gaussian distribution with a triangle function, with the beam influence decreasing from its maximum at the center ray to zero at the adjacent rays. The result is a model that precisely recovers the ray-theoretic result. Weinberg and Keenan [5] introduced the similar concept of Gaussian Ray Bundles (GRB), but with an additional feature which limits the beam focusing such that simple caustics are handled accurately. Without these limits, GRB and GBT are essentially the same, except for beam distribution used. Both the SGB and Cerveny methods require certain parameters to be set (e.g. the starting conditions of the beams) to run the models. This is not the case for the GBT and GRB approaches, which is an obvious advantage.

The purpose of this paper is to compare predictions obtained by these methods with each other and a reference solution (normal modes). Such a comparison is useful for gaining an understanding of the merits and drawbacks of each approach.

2. BEAM TRACING ALGORITHMS

All algorithms trace a fan of rays from a source using the standard ray equations [6]:

$$\frac{dr}{ds} = cx(s), \quad \frac{dz}{ds} = cz(s), \quad \frac{dx}{ds} = -\frac{1}{c^2}\frac{dc}{dr}, \quad \frac{dz}{ds} = -\frac{1}{c^2}\frac{dc}{dz}, \tag{1}$$

where *r(s), z(s)* is the ray trajectory in cylindrical coordinates, ($x(s)$,$z(s)$) is a tangent to the ray, *s* is arc length along the ray, and *c(s)* is the sound speed along the ray. The initial conditions specify the source location and the slope of the emitted ray:

$$r(0) = r_0, \quad z(0) = z_0, \quad x(0) = \frac{\cos a}{c(0)}, \quad z(0) = \frac{\sin a}{c(0)}, \tag{2}$$

where (r_0, z_0) is the source coordinate and a is the ray launch angle.

The pressure field due to each beam can be written as:

$$P(s,n) = A(s) f(n,s) e^{iwt}, \tag{3}$$

where w is the source frequency, *t(s)* is the travel time along the ray, *A(s)* is the amplitude along a ray, *n(s)* is the normal distance from the receiver to the central ray of a beam, and *f(n)* is the influence function in the direction normal to the ray path. The four beam tracing algorithms differ mainly in the choice of *A* and *f*. The expressions for *A* and *f* for the four algorithms are simply presented here for a point source; the reader is referred to references [1]-[5] for derivation details. In these expressions, *r* is the range, da is the difference in angles between adjacent rays, and *W(s)* is the beam-width in the direction normal to the ray.

Cerveny:

$$f(n,s) = \exp\left\{-0.5iw\left(\frac{p(s)}{q(s)}\right)(n^2)\right\}, \quad A(s) = \left(\frac{da}{c(0)}\right)\exp\left(\frac{ip}{4}\right)\sqrt{\frac{q(0)w\cos a}{2p}}\sqrt{\frac{c(s)}{rq(s)}}, \tag{4}$$

where *p(s)* and *q(s)* determine the beam curvature and width, and are obtained by integrating a pair of ordinary differential equations along the central ray (see Ref. [1]). The initial values of *p* and *q* must be selected. In this paper, it is assumed that the beam is initially flat and that the initial beam-width is such that the beams are "space filling" in the farfield [$p(0)=1$, $q(0)=2ic^2(0)/\{w(da)^2\}$].

SGB:

$$f(n,s) = \exp\left\{-a\left(\frac{n}{s}\right)^2\right\}, \qquad A(s) = \frac{da}{s}\sqrt{\frac{a}{p}\cdot\cos a}, \tag{5}$$

where $a = -4\ln(b)/(da)^2$. b, called the beam factor, is the value of the beam at the midpoint between adjacent beams, and is specified by the user. In actual implementation, a correction factor is included in *f* to account for wavefront curvature.

GBT:

$$f(n,s) = \begin{cases} \dfrac{W(s)-n}{W(s)} & for \;\; n \le W(s) \\ \\ 0 & else \end{cases} \quad , \quad where \quad W(s) = \left| \frac{q(s)da}{c(0)} \right| \tag{6}$$

$$A(s) = \sqrt{\frac{c}{c(0)} \cdot \frac{da}{r} \cdot \frac{\cos a}{W}} \; .$$

GRB:

$$f(n,s) = \exp\left\{ -\left(\frac{n}{W} \right)^2 \right\}, \qquad A(s) = \frac{1}{(2p)^{1/4}} \sqrt{\frac{c}{c(0)} \cdot \frac{da}{r} \cdot \frac{2\cos a}{W}} \; , \tag{7}$$

where $W = q(s)da/c(0)$. The implementation of GRB in this paper differs from that of Weinberg and Keenan [5] in that beams are formed in terms of pressure instead of intensity.

3. TRANSMISSION LOSS COMPARISONS FOR TEST CASE

The test case considered in this paper is a deep-water (5000 m) environment with the canonical Munk profile, which is plotted in Fig. 1 along with the corresponding ray trace for a source depth of 1000 m. The rays shown are for launch angles between –20° and 20° with a 1° increment. Figure 2 presents the KRAKEN normal mode[7] solution for the transmission loss for the same source depth and launch-angle limits at a frequency of 50 Hz. For the KRAKEN result, the bottom was assumed to be a homogeneous fluid half-space with a compressional sound speed of 1600 m/s, a density of 1.0 g/cm^3, and no volume attenuation. All of the beam-tracing results will also use this bottom model. The field is seen to exhibit a typical deep-water convergence-zone behavior, containing several caustics and shadow zones. Some limited bottom-reflecting energy is also included, causing energy to partially fill the shadow zones.

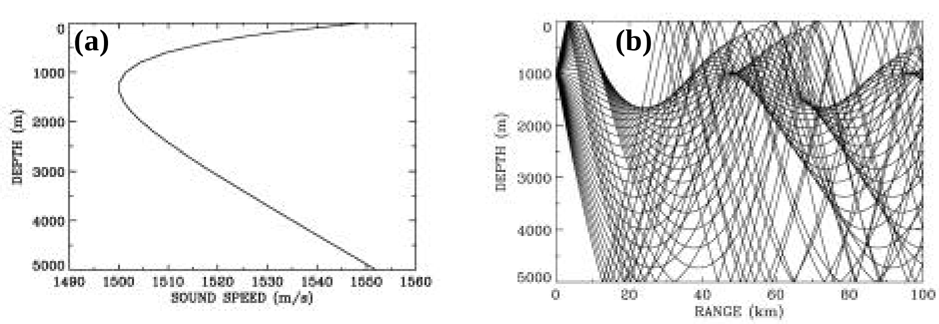

Fig. 1. (a) Munk sound speed profile and (b) corresponding ray trace for a source depth of 1000 m. Launch angles in (b) are between –20° and 20° with a 1° increment.

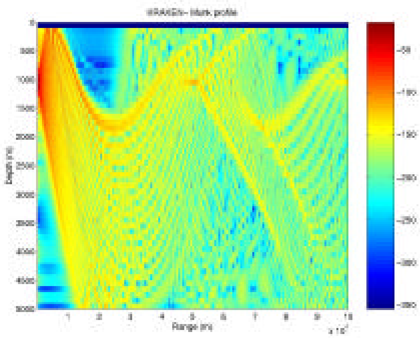

Fig. 2. 50-Hz transmission loss for the Munk profile using KRAKEN normal mode code. Source depth = 1000 m. Launch-angle spread is –20° to +20°.

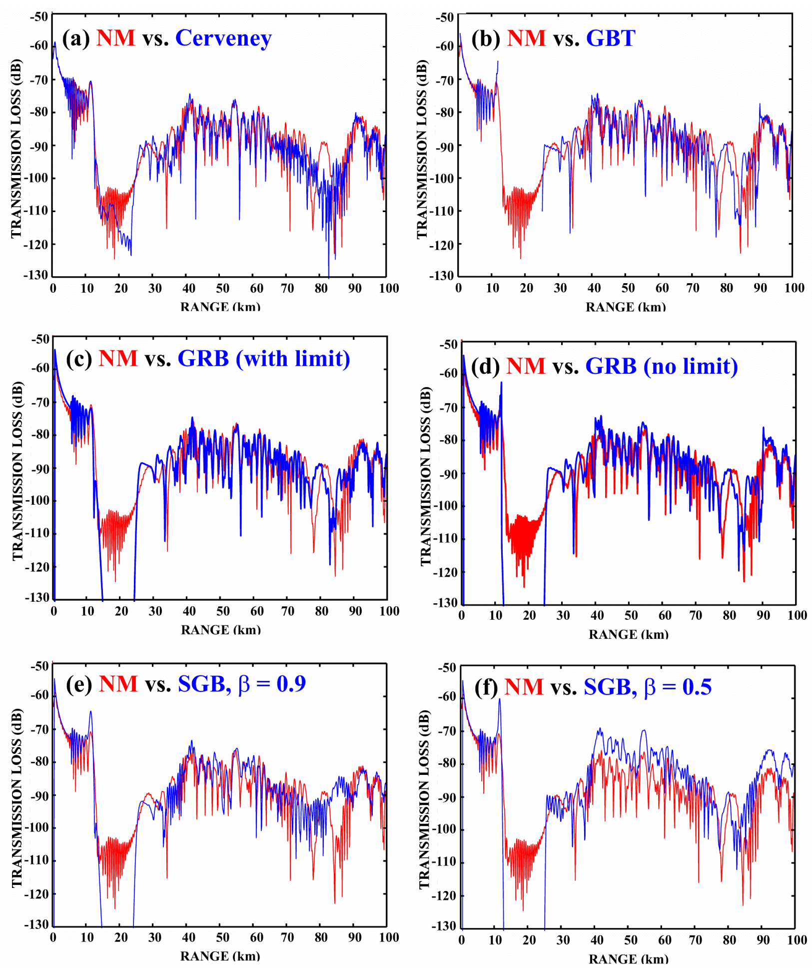

Fig. 3. Comparison of predicted 50-Hz transmission loss using beam-tracing algorithms (blue curves) with reference normal mode solution (red curve). Source depth = 1000 m. Receiver depth = 800 m. 100 beams launched between –20° and +20°.

The 50-Hz transmission loss predicted by each of the four beam-tracing algorithms (blue lines) for the test case is compared with the corresponding KRAKEN normal mode result (red line) in Fig. 3. The source depth is 1000 m, the receiver depth is 800 m, and 100 beams were launched between –20° and 20° in all cases. Figure 3a is for the Cerveny method with the optimal starting beam conditions, Fig.3b for GBT, Fig. 3c for GRB with beam-focusing limits, Fig. 3d for GRB without beam-focusing limits, Fig. 3e for SGB with a beam factor of 0.9, and Fig. 3f for SGB with a beam factor of 0.5. All beam tracing algorithms are seen to agree well with the normal mode reference solution, with the exception of the SGB result for a 0.5 beam factor, where the resulting beam-widths are too large. (The results between roughly 12 and 23 km are not meaningful since this is the portion of the shadow zone without bottom-reflected energy.) The Cerveny method has difficulty reproducing the peak in the normal mode solution near 80 km while SGB with a 0.9 beam factor is unable to predict the valley in the normal mode solution near 85 km. Sharp caustics are present in the GBT and GRB/no beam-focusing-limit results, since the methods basically recover the ray-theoretic result. The effectiveness of the GRB beam-focusing limits in suppressing caustics is demonstrated by comparing Fig. 3c with Fig. 3d.

4. SUMMARY

The similarities and differences between four beam-tracing algorithms have been discussed and the quality of their predictions compared for a deep-water test case. Other test cases are presented in the conference presentation. All algorithms agree well enough with the reference solution to warrant their use in practical applications. GBT and GRB possess the important advantage of not requiring certain parameters to be set before execution. The beam-focusing limits with GRB provide useful means of handling caustics.

5. ACKNOWLEDGEMENTS

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