Proceedings of the 12th World Congress on Scientific Computation ed. R Vichnevetsky et al (1988)

RAPID THREE-DIMENSIONAL OCEAN ACOUSTIC MODELING OF COMPLEX ENVIRONMENTS

W.A. Kuperman, M. B. Porter, J. S. Perkins and A. A. Piacsek [1]
Naval Research Laboratory, Washington, DC 20375

A wave-theory based technique for the rapid computation of propagation loss over a wide geographical area with a complex 3D environment has been developed. The method relies on an adiabatic normal mode approach that reduces the wave equation to a set of coupled equations in the depth variable z and the plan variables (x, y). A unique feature of the technique which greatly speeds the computation, is the precalculation of an impedance surface that replaces the oceanographically stable portion of the sound-speed profile and the geo-acoustic structure of the ocean bottom, both derived from archival information.

Introduction

The computational issues central to three-dimensional (3D) ocean acoustic modeling have been highlighted in previous papers [1,2,3]. Much of these issues arise from the "marching" nature of the algorithms employed. That is, different source/receiver configurations relative to the environment require recomputation of the total acoustic wave equation solution. Here we briefly present an approximate technique [4] based on mode theory to rapidly compute the acoustic field for complex 3D ocean environments in a nonredundant fashion. The primary goals of this research are to investigate the graphical patterns that emerge for complex environments where the acoustic field reflects or "images" the oceanography and topography and to provide a method to generate acoustic "data" for sophisticated numerical experiments. Furthermore, it is our belief that these 3D representations using color graphics [4] to display and convey complex ocean acoustic phenomena will provide hints and an impetus for developing new approaches to the 3D modeling problem.

Traditionally ocean acoustic modeling has been done assuming a cylindrically symmetric environment. On the other hand, 3D variation of sound speed, bathymetry and bottom type is common in the ocean. Seamounts and mesoscale eddies are just two examples of features which disturb the cylindrical symmetry of the problem. With regard to such features, we make the following observations: topographical features are stationary and even volume medium effects via the sound speed profile are unchanging below certain depths known to oceanographers. The 3D ocean acoustic environment is then relatively benign to a great extent which in itself suggests that we somehow seek a computational technique which takes advantage of the stability of the environment. An adiabatic normal mode procedure is used as a basis to meet the criteria of taking advantage of the local stability of the medium in order to contruct 3D "acoustic images" of the environment. Horizontally, we grid the ocean environment in terms of its local acoustic eigenvalues and normal modes. We further take advantage of the deep stability of the ocean in the mode calculation by precalculating an impedance function by numerically integrating the local depth normal mode equation up from the basement to a surface below which the oceanography is stable. This complex impedance function at the upper boundary of the stable part of the water column is, then, a composite of the local bottom type, bottom depth and complete sound speed profile below this impedance surface. The stored horizontal grid of impedance functions then represents the precomputed part of a 3D acoustic wave equation solution descriptive of a large ocean region which is computationally "shallow" since it is now bounded in depth by the impedance surface. The local modes, if not already known (for example if the upper profile is known we take the impedance surface to be the pressure release ocean surface and then we might as well just store the modes), are finally obtained by "shooting" down from the ocean surface to the impedance surface. For mild horizontal variability, we construct the 3D acoustic field by a straightforward adiabatic mode computation using the gridded modal information. For stronger horizontal variability, we use a horizontal grid of the eigenvalues for each mode to construct a horizontal modal sound speed structure in order to calculate the horizontally refracted normal mode field using a Gaussian beam algorithm [5].

The most important aspect of this procedure is that most of the computation is independent of the specific source/receiver configurations allowing changing environment relative to source/reciever configuration to be recalculated with minimum effort as opposed to the total recalculation necessary in any marching algorithm. Speed up over conventional marching methods is then accomplished by using this "spreadsheet" type approach of manipulating wave equation precalcuations. The speed up is particularly effective in the majority of cases where horizontal refraction computations [1] need not be performed.

Formulation

Our starting point is the Helmholtz equation in three-dimensions. We have

$$\nabla^2 p + \frac{\omega^2}{c^2(x,y,z)} p = \delta(x)\delta(y)\delta(z - z_s). \qquad (1)$$

Here, ω is the circular frequency of the source, $c(x, y, z)$ is the ocean sound speed and $p(x, y, z)$ is the acoustic pressure. The normal mode solution obtained when $c(x, y, z) = c(z)$, i.e. the case of a stratified cylindrically symmetric ocean is well know and is extended to mild range dependence using the adiabatic approximation.

Adiabatic mode equations

The adiabatic *range-dependent* normal mode result is [4]

$$p(r, z) = \sum_{j=1}^{m} u_j(z_s; 0) u_j(z; r) \frac{e^{i\int_0^r k_j(r) dr}}{\sqrt{k_j(r) r}}, \qquad (2)$$

where $u_j(z; r)$ satisfies,

$$\frac{d^2}{dz^2} u_j(z; r) + \left(\frac{\omega^2}{c^2(z; r)} - k_j^2\right) u_j(z; r) = 0, \qquad (3)$$

$$u_j(0; r) + Z^T(k_j^2) \frac{du_j}{dz}(0; r) = 0, \qquad (4)$$

$$u_j(D; r) + Z^B(k_j^2) \frac{du_j}{dz}(D; r) = 0. \qquad (5)$$

Thus, the modal sum involves the *local modes* $u_j(z; r)$, that is the modes calculated using the sound speed profile at the receiver range, r. Similarly, the mode excitation coefficients, $u_j(z_s; 0)$ use the modes calculated using the sound speed profile at the source range, $r = 0$.

The 3D adiabatic mode equations

The above solution makes use of modes $(u_j(z; r), k_j(r))$ as if they were available at a continuum of range points. In practice, the computation of modes can be somewhat computationally expensive and it is desirable to compute local modes at as few range slices as possible. Thus, the environment is subdivided at points $r = r_m, m = 1, \ldots, N_{prof}$ where N_{prof} denotes the number of profiles. Then one solves for a set of modes at each r_m and uses linear interpolation to construct modes at ranges which lie between those r_m where the modes have been calculated.

[1] Planning Systems Inc., McClean, VA 22102

We may now describe the $Nx2D$ generalization for a 3D environment. Basically, the procedure is to solve the 3D problem on N azimuthal slices using range-dependent (adiabatic) mode theory. Each azimuthal slice is treated as an independent modal problem as if each slice of the sound speed profile and bathymetry profile were derived from a cylindrically symmetric problem. This approach has also been applied in parabolic equation modeling [1]. The normal mode result is,

$$p(r,z,\theta) = \sum_{j=1}^{m} u_j(z_s;0,0)u_j(z;r,\theta)\frac{e^{i\int_0^r k_j(r,\theta)dr}}{\sqrt{k_j(r,\theta)r}}, \tag{6}$$

where,

$$\frac{d^2}{dz^2}u_j(z;r,\theta) + (\frac{\omega^2}{c^2(z;r,\theta)} - k_j^2)u_j(z;r,\theta) = 0, \tag{7}$$

$$u_j(0;r,\theta) + Z^T(k_j^2)\frac{du_j}{dz}(0;r,\theta) = 0, \tag{8}$$

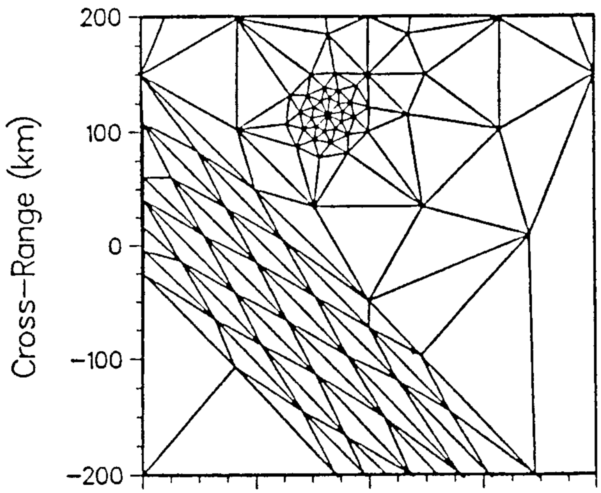
$$u_j(D;r,\theta) + Z^B(k_j^2)\frac{du_j}{dz}(D;r,\theta) = 0. \tag{9}$$

seamounts or continental slopes they may also be deflected out of the incident vertical plane.

The inclusion of horizontal refraction into a modal formulation has been previously discussed by Pierce [2] and Weinberg and Burridge [3]. Our approach [4], however, is substantially different; we employ Gaussian beam tracing [5] to solve the lateral wave equation which eliminates both the problem of finding eigenrays and the problem of including caustic or shadow-zone corrections. The Gaussian beam approach applied to the horizontal refraction problem is discussed in detail in [4]. Essentially, the horizontal field of each modal wavenumber is translated into a horizontal sound speed field which defines the Gaussian beam environment for each mode.

Precalculations for the mode computation

Since speed of computation is central to 3D modeling, we have also found it advantageous to reduce the solution of the modal equations (Eqns. 7 and 9) to a "precalculated" part corresponding to the local unchanging part of the ocean and a final mode computation which interfaces the precalculated modal information with a mode computation involving the latest update to the upper part of the water column. The precalculated modal information will be in the form of an impedance function obtained by integrating the eigenvalue equation (parametrized by horizontal wavenumber) from the basement to the uppermost stable region of the water column. Therefore, for calculations in which the local environment will itself be variable, it will be the impedance function that will be stored at the above mentioned nodes rather than modes thereby allowing for extremely rapid updating of the local environment which is ultimately included in the 3D computation. This procedure eliminates redundant calculation in many cases where the sound speed, for example, is changing in only the upper few percent of the water column. This procedure is illustrated in Fig. 2 and the details of the construction of this surface, whose depth is determined by the local bathymetry and oceanography, is presented in detail in [4]. The total numerical eigenvalue algorithm that is used is based on the work of Porter and Reiss [6].



three-dimensional structure of the water column and ocean bottom. To demonstate the wide area capability of the procedure presented in this paper we present results for sound propagation in a "Generic Sea" which by construction contains three-dimensional variability in the water column in the form of a warm core eddy and bottom structure in the form of a seamount and a continental shelf.

Figure 1 is the top view of the grid structure of a "Generic Sea" containing features of a 3D range-dependent environment. We have arbitrarily picked a 400 km square to compute the three dimensional acoustic field. Contained in this ocean is a range indepenent deep water region of 5000 m depth, a continental slope rising to a shelf of 1000 m depth, a seamount and a "warm core" eddy overlapping the seamount of radius 200? km as depicted by the series of sound speed profiles in Fig. 2.

The $N \times 2D$ computation for this environment is shown in Fig. 3 (The gray-scale in Fig. 6 refers to Figs. 3-5.) and can be interpreted in two ways (Ref. [4] presents Figs. 3-5 in color contours where the structure of the acoustic field is much more clearly evident.):

1. It is a contour of transmission loss for a source at range zero and depth 100 m over a receiver plane at depth 160 m. Figure 4 which is a vertical slice of the three dimensional field emanating due east from the center point of Fig. 3 plane shows clearly that the outer white rings are convergence zones and the two inner white rings are from bottom-bounce paths.

2. Conversely, Fig. 3 can be interpreted as a contour of transmission loss to a receiver at the center point (depth 100)m from sources at all points in the plane at depth 160 m. For example, due east from the center at range 200 km, the white ring indicates that a source at depth 160 m will propagate by a convergence zone path to the center receiving point.

The 3D computation with horizontal refraction is shown in Fig. 5 where we see the additional shadowing beyond the seamount caused by the acoustic paths bending away from the sides of the seamount.

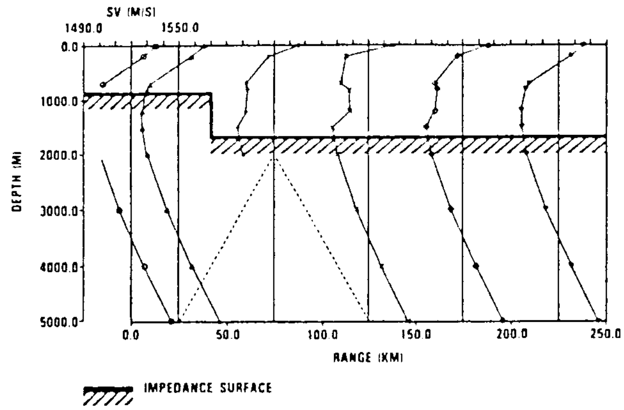


Figure 2: Impedance surface. The ocean is taken to be stable below the impedance surface. The depth of the surface is determined by the local oceanography. The acoustic information stored at a node contains all the stable modal search information incorporating the local unchanging parameters: bottom depth, bottom type, sound speed profile up to the impedance surface.

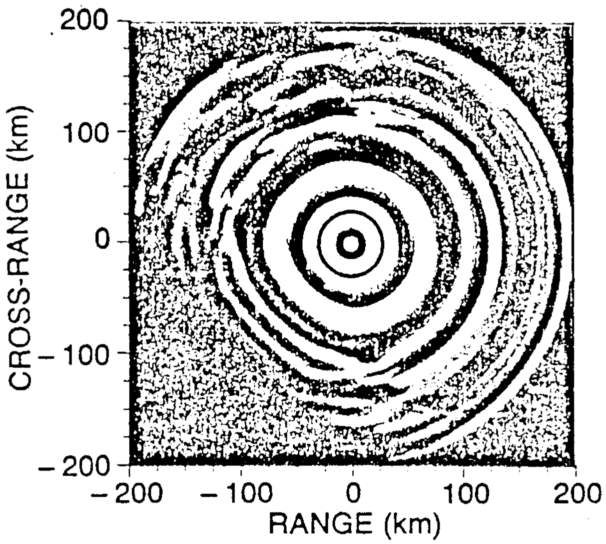


Figure 3: Generic Sea. Contours of transmission loss for the environment imposed on the grid in Fig. 1.

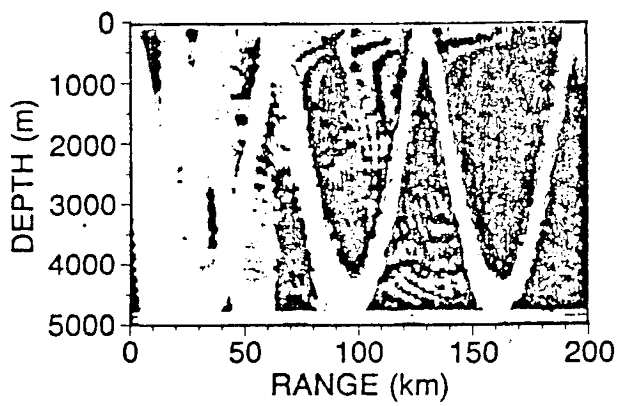


Figure 4: Generic Sea. Contours of transmission loss for a vertical slice of the environment due east of the center of Fig. 3.

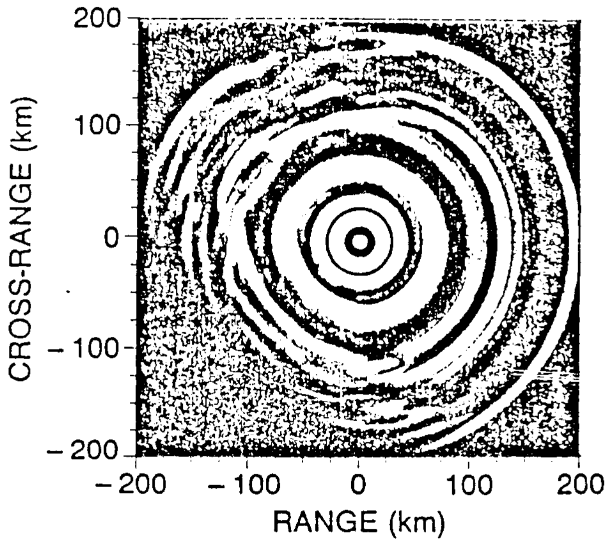


Figure 5: Generic Sea. Transmission loss contours over the same region as in Fig. 3 but with horizontal refraction included.

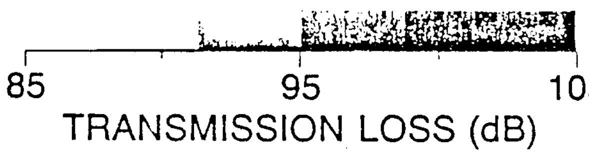


Figure 6: Gray-scale levels used in all the contour plots.

Summary

A combination of displays such as those in Figs. 3 and 4 is, in essence a 3D acoustical "image" of the ocean. It is also clear that these figures demonstrate the capability of the technique to produce numerical acoustical "data" for a large oceanographic area. Excluding precalculation time, snapshots of the acoustic field can be computed in about 3 minutes on a VAX 11/780 for a particular source location. A remaining issue is that of the need for mode coupling, i.e., the shortcomings of the adiabatic approximation. While we have not yet included this feature, it is straightforward to do so. At this point, it is not clear whether (or in which cases) horizontal refraction or mode coupling is more important.

References

[1] John S. Perkins and Ralph N. Baer,"An approximation to the three-dimensional parabolic-equation method for acoustic propagation", J. Acoust. Soc. Am. 72:515–522 (1982).

[2] Allan D. Pierce, "Extension of the method of normal modes to sound propagation in an almost-stratified medium," J. Acoust. Soc. Am., 37:19–27 (1965).

[3] Henry Weinberg and Robert Burridge, "Horizontal ray theory for ocean acoustics," J. Acoust. Soc. Am. 55:63–79 (1974).

[4] W. A. Kuperman, Michael B. Porter, John S. Perkins and Andrew A. Piacsek, " Rapid computation of acoustic fields in three-dimensional environments," submitted for publication in J. Acoust. Soc. Am.

[5] Michael B. Porter and Homer P. Bucker, "Gaussian beam tracing for computing ocean acoustic fields", J. Acoust. Soc. Am. 82:1349–1359 (1987).

[6] Michael B. Porter and Edward L. Reiss, "A numerical method for bottom interacting ocean acoustic normal modes", J. Acoust. Soc. Am. 77:1760–1767 (1985).